Development of biomechanical gait analysis based on inverted pendulum theory

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Abstract. Systems based on inverted theory pendulum are among the most difficult to adjust due to its instability. However, human body balances this type of system daily, from simple movements to the most complex. This paper present a type of biomechanical analysis of human gait using inverted pendulum theory. In the introduction are presented general notions used for the analysis of gait and bipodal mechanism. The next chapter presents reverse pendulum theory by means of a simple application, that of a trolley and a pendulum which seeks its equilibrium position when a force is applied on the trolley. By the action of the force, the pendulum finds its equilibrium position but which disappears when the cart stopped. Chapter three presents the experimental application of gait analysis using static images. The last chapter represents the conclusions of this type of analysis.

1. Introduction

The human locomotor system consists of bones, joints, ligaments and muscles that represent the basic system that leads to movement. The movement is done in the joints by their property of contracting. The lower limb is the body for maintaining the body in a vertical position, support and for bipedal walking. Conventional gait analysis divides the body into 7 segments (pelvic girdle, hip, thigh, knee, leg, ankle, leg) which are connected by joints. [1]

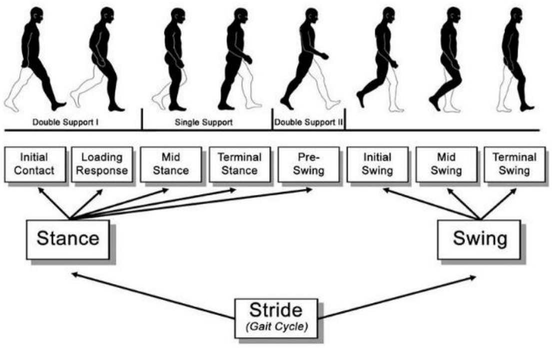

Figure 1. Walking cycle [6]

Stabilizing the system for a reverse pendulum is an example that the human body accomplishes every day. The manual application of the go-to-come movement is the most common for balancing the reverse pendulum system attached to a stroller. Instead, the human body regulates this balance through muscle stiffness (more precisely, ankle stiffness). Compared to the problem illustrated in chapter two, in the case of the bipodal support position there is only one degree of freedom, namely the ankle joint. [4].

1.1. Bipodal mechanism

During the bipedal position, the femur transmits the weight of the body to the leg. When the lower limb is not in contact with the ground, the femur is like a third-degree lever having a point of support in the hip joint, muscle forces being applied to the middle of the femur, and on contact with the ground it becomes a first-degree lever with the point of support. on the transverse axis of the knee. [5] Therefore the gait is divided into the support phase (bipodal support) and the balance phase (in which one leg becomes support and the other becomes motor). The walking phases are shown in Figure 1.

In the balance phase, the foot behaves like an inverted pendulum in which the supporting position is the ankle joint, and the tibia is the pendulum, ensuring the movement together with the knee joint.

2. Inverted pendulum theory

The inverted pendulum represents an instable system that has it's center of gravity upper than the pivot position [2]. In order to be exemplified, the following problem is taken (Figure 2). It is unstable without control, meaning the pendulum drops if the stroller is not moved to balance it. The dynamics of the system are nonlinear, and the objective of the control system is to balance the reverse pendulum by applying a force on the trolley to which the pendulum is attached.

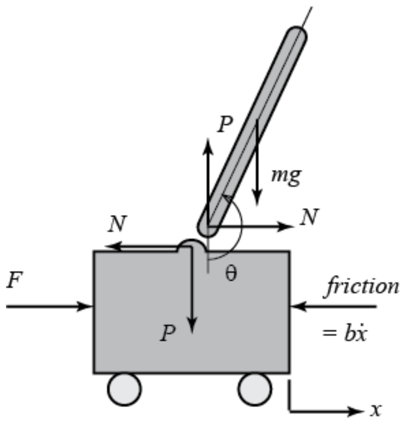

Figure 2. Analyzing the inverse pendulum[3], where: M – mass of the cart; m –mass of the pendulum; l – length to pendulum center of mass; I – mass moment of inertia of the pendulum; F – force applied to the cart; x- cart position coordinate; θ – pendulum angle from vertical (down)

2.1. *Developing the equations of movement*

Movement equation for horizontal direction is obtained by summing forces (Figure 1) resulting:

$$M\ddot{x} + b\dot{x} + N = F, \text{ b – coeffcient of friction for cart.} \qquad (2.1.1.)$$

Force of reaction N is obtained by adding forces on horinzontal direction resulting equation (2.1.2) [3], which replacing it in (2.1.1) [3], results in equation (2.1.3) [3] which represent the first equation of movement of the control system.

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \qquad (2.1.2.)$$

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \qquad (2.1.3.)$$

Second equation of movement is obtained by summing perpendicular forces from the pendulum (2.1.4) [3].

$$\mathrm{P}\sin\theta + \mathrm{N}\cos\theta - \mathrm{mg}\sin\theta = \mathrm{ml}\ddot{\theta} + \mathrm{m}\ddot{x}\cos\theta \qquad (2.1.4)$$

In order to eliminate P and N from the equation, it is needed to sum the momentum in the center of gravity of the pendulum obtained equation (2.1.5) [3] which replacing it in equation (2.1.4) [3] results the second equation of movement of the control system (2.1.6) [3].

$$-\mathrm{Pl}\sin\theta - \mathrm{Nl}\cos\theta = \mathrm{I}\ddot{\theta} \qquad (2.1.5)$$

$$(\mathrm{I} + \mathrm{ml}^2)\ddot{\theta} + \mathrm{mgl}\sin\theta = -\mathrm{ml}\ddot{x}\cos\theta \qquad (2.1.6)$$

The system composed of equations (2.1.3) [3] and (2.1.6) [3] need to be linearized in order to determine the transfer functions. Thus we assume that the angle θ is equal to π and that we have a maximul deviation of 20° from the vertical position that we represent with Φ. This results in about -1, sin θ = -Φ, and θ˙^2 = ϕ˙^2≈0. After replacing these approximations of nonlinear functions in our system of equations we obtain the two linearized equations, and the force F is replaced by u.

$$(\mathrm{I} + \mathrm{ml}^2)\ddot{\phi} - \mathrm{mgl}\phi = \mathrm{ml}\ddot{x} \qquad (2.1.7)$$

$$(M + m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u \qquad (2.1.8)$$

2.2. Transfer functions of the system

The function of transfer of the linearized equations system are obtained using Laplace considering zero initial conditions. The resulting Laplace transforms is equation 2.2.1 and 2.2.2 [3]:

$$(\mathrm{I}+\mathrm{ml}^2)\phi(\mathrm{s})s^2-\mathrm{mgl}\phi(\mathrm{s})=\mathrm{mlX(s)}s^2 \quad (2.2.1)$$

$$(M+m)X(s)s^2+bX(s)s-ml\phi(s)s^2=U(s) \quad (2.2.2)$$

In order to eliminate function X(s) we replace in equation (2.10) [3] and we obtain:

$$(M+m)\left[\frac{I+ml^2}{ml}-\frac{g}{s^2}\right]\phi(s)s^2+b\left[\frac{I+ml^2}{ml}-\frac{g}{s^2}\right]\phi(s)s-ml\phi(s)s^2=U(s) \quad (2.2.3)$$

Rearranging the above equation and rewriting for origin 0 and replacing in the second equation of the system results the following equatios (2.2.4 and 2.2.5) [3]:

$$P_{pend}(s)=\frac{\phi(s)}{U(s)}=\frac{\frac{ml}{q}s}{s^3+\frac{b(I+ml^2)}{q}s^2-\frac{(M+m)mgl}{q}s-\frac{bmgl}{q}} \qquad \left[\frac{rad}{N}\right] \quad (2.2.4)$$

$$P_{cart}(s)=\frac{\mathrm{X(s)}}{U(s)}=\frac{\frac{(I+ml^2)s^2-gml}{q}}{s^4+\frac{b(I+ml^2)}{q}s^3-\frac{(M+m)mgl}{q}s^2-\frac{bmgl}{q}s} \qquad \left[\frac{m}{N}\right] \quad (2.2.5)$$

where $q=[(M+m)(I+ml^2)-(ml^2)]$ (2.2.6)

The equations obtained this way (2.2.4 and 2.2.5) [3] are introduced in software application MATLAB in order to be calculated automatically. This way, knowing the initial data of the system that needs to be adjusted will result position of the pendulum and the cart. These determinations is the theoretical procedure for evaluation the way in which the walking cycle is performed in relation to the contact of the foot with the ground (pendulum-cart joint).

3. Development of the experimental evaluation system of the locomotor system movement

The experimental part was realized on a male subject, 29 years old, without locomotor problems, who was asked to perform a few steps to exemplify the movement of the locomotor system. In each of the required positions, the lengths of the segments between the joints and the angle between the two were measured.

The lengths of the segments were measured with an anthropometric instrument for linear dimensions, these being 510 mm for the distance between the hip joint and knee joint, and 440 mm for the distance

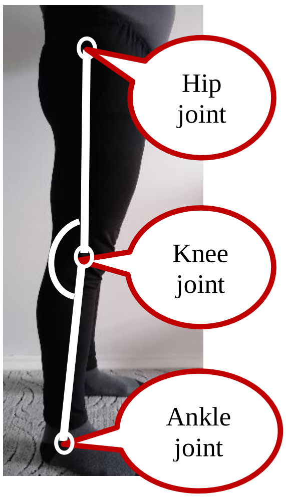

Figure 1.a. Initial position – beginning of the walking cycle

Figure 1.b. Measuring with the mechanical goniometer

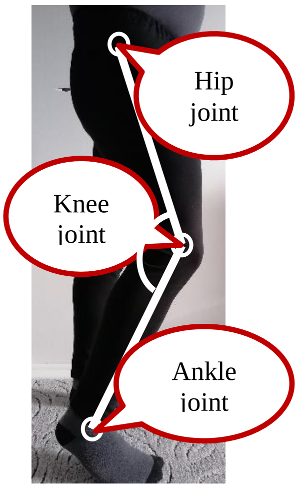

Figure 2. a. Position 2 – left leg support, detachment right leg

Figure 2.b. Measuring with the mechanical goniometer

between knee joint and ankle joint. These length do not change over the walking cycle. The angle between these two segments that forms the knee joint was measured for every step from the walking cycle with a mechanical goniometer, and the values were mentioned in each position (Figure 1b-7b).

Bipodal position is when the both legs are in contact with the ground and the weight is distributed evenly on both legs. In the initial position (Figure 1a) was measured an angle of 175° (Figure 1b) meaning that the subject has the right knee slightly bend. Next step is the position when the right leg is displaced from the ground, and the left leg is the support (Figure 2a), where the angle between the two segments is 128° (Figure 2b). Position 3 is characterized by the right leg that touches the ground completely becoming the support for the walking and left leg that is detached from the ground (Figure 3a). In this step the angle between segments is 180° (Figure 3b), meaning that the right leg is stretched and in tension. Figure 4a presents the next bipodal support where both legs are on the ground, with an angle of 175° (Figure 4b), where the right knee is slightly bend.

Figure 3. a. Position 3- right leg support, left leg detachment

Figure 3.b. Measuring with the mechanical goniometer

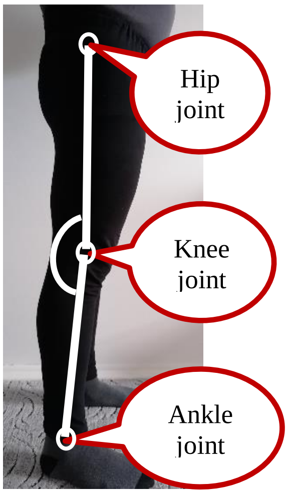

Figure 4.a. Position 4 – bipodal support

Figure 4.b. Measuring with the mechanical goniometer

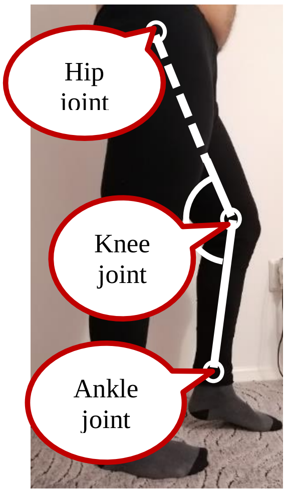

Figure 5.a. Position 5 – left leg detachment, right leg support

Figure 5.b. Measuring with the mechanical goniometer

Figure 6.a. Position 6 – left leg support, right leg detachment

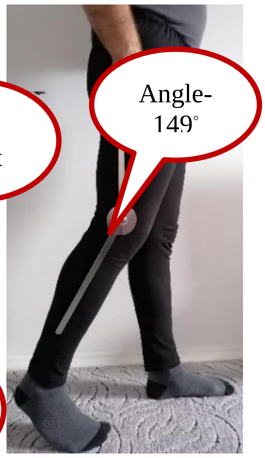

Figure 6.b. Measuring with the mechanical goniometer

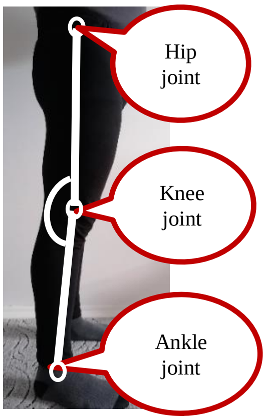

Figure 7.a. Final position – bipodal support

Figure 7.b. Measuring with the mechanical goniometer

The fifth position presents the left leg that detached from the ground and the right leg became the support (Figure 5a). In this case the angle between the two segments is 130° (Figure 5b) meaning that this leg is bending a little bit more than the right leg. Position 6 (Figure 6a) is similar with the second one, the only exception is the angle measured at 149° (Figure 6b), resulting a difference of 21° between the two positions. The final position (Figure 7a and 7b) is the same with the first one, meaning that the body is in the same position and the cycle of walking is complete.

4. Conclusions

The reverse pendulum is illustrated in the walking cycle by the ankle joint which is comparable to the joint between the trolley and the pendulum. The angle between segment one (measured between hip joint and knee joint) and segment two (measured between knee joint and ankle joint) illustrates the needs to identify the equilibrium position of the human body from a lower angle to a larger one and again a small angle. This is due to the reverse pendulum theory which finds its equilibrium position when walking in order to maintain the body upright.

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